pulse, it is possible to produce a pseudorandom signal coinciding with the clocking rate of $M_t$. This fact makes it possible to achieve output rates of the order of $10^9$ numbers/s.

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References


Slant Haar Transform

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Abstract—The slant Haar transform (SHT) is defined and related to the slant Walsh–Hadamard transform (SWHT). A fast algorithm for the SHT is presented and its computational complexity computed. In most applications, the SHT is faster and performs as well as the SWHT.

Introduction

The slant Walsh–Hadamard transform (SWHT) (originally called slant transform) has been proposed by Enomoto and Shibata [1] for the order 8 and used in TV image encoding. Pratt et al. [2] and Chen [3] have generalized this transform to any order $2^n$ and compared its performance with other transforms. In [4], we have given a simpler definition of the SWHT as a particular case of a unifying treatment of fast unitary transforms and computed the number of elementary operations required by its fast algorithm. The interesting feature of the SWHT is the presence of a slant vector with linearly decreasing components in its basis. On the other hand, we have found that locally dependent basis vectors, such as in the Haar transform (HT), are of interest [5]. In this letter, we define a composite fast unitary transform: the slant Haar transform (SHT). We show that its relations to the SWHT parallel the relations between the HT and the Walsh–Hadamard transform (WHT) [6]. This previous work leads us to expect that the SHT has an advantage over the SWHT because of its speed and comparable performance.

Definition

The generalized Kronecker product of the set $\{\mathcal{A}\}$ of $n$ matrices $A^j$ ($j = 0, \ldots, n-1$) of order $m$ and the set $\{\mathcal{B}\}$ of $m$ matrices $B^k$ ($k = 0, \ldots, m-1$) of order $n$ is the matrix $[\mathcal{C}]$ of order $mn$ such that $C_{mn+jk} = A_{mj}B_{nk}$ where $A, B < n$ and $m < n$. $[\mathcal{C}]$ can be factorized and has a fast algorithm [4]. The generalized Kronecker product provides a simple way to recursively define fast unitary transforms. Consider the matrix of order 2:

$$[T_2] = [F_2(x/4)] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Then the matrix of order $2^n$, denoted $[T_{2^n}]$, is obtained from the matrix of order $2^{n-1}$ by $[\mathcal{A}] \otimes [T_{2^n-1}]$ where $\otimes$ denotes a generalized Kronecker product. With this recursive notation, the HT is obtained for $\{\mathcal{A}\} = [F_2(x/4)]$, $[I_2]$, $\ldots$, $[I_2]$ and the WHT for $\{\mathcal{A}\} = [F_2(x/4)]$, $[I_2]$, $\ldots$, $[I_2]$ and the WHT.

Relations between SHT and SWHT

The SHT and SWHT have relations similar to the relations between the HT and the WHT [6]. Partition the ordered SHT (SWHT) into $2n$ rectangular submatrices, denoted $[\text{MSH}_{\alpha,\beta}^{k,\ell}][\text{MSH}_{\alpha,\beta}^{k,\ell}]$, $k = 0, \ldots, n-1$ and $\ell = 0, 1$. For $k, \ell = 0, 1$, $[\text{MSH}_{\alpha,\beta}^{0,0}]$, $[\text{MSH}_{\alpha,\beta}^{0,1}]$, $[\text{MSH}_{\alpha,\beta}^{1,0}]$, and $[\text{MSH}_{\alpha,\beta}^{1,1}]$ are the first four rows of the ordered matrix $[\text{SHT}_{\alpha,\beta}]$. For $k > 1$, $[\text{MSH}_{\alpha,\beta}^{k,\ell}]$ is formed from the rows of rank $2^{k-1}+i2^{k-2} < \ell < 2^{k-1}+i2^{k-2}$. The matrices $[\text{MSH}_{\alpha,\beta}^{k,\ell}]$ are similarly defined. The submatrices for the order 8 are shown in Fig. 1. It can be shown, following the proof given in [6], that

$$[\text{MSH}_{\alpha,\beta}^{k,\ell}] = [S_{\alpha,\beta}] [\text{WHT}_{\alpha,\beta}] [\text{MSH}_{\alpha,\beta}^{k,\ell}]$$

where

$$[S_{\alpha,\beta}] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 7 & 5 & 3 & 1 & -1 & -3 & -3 & -3 \\ 3 & 1 & -1 & 3 & -3 & -1 & 3 & 1 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -3 & 3 & -1 & 0 & 0 & 0 & 0 \\ 5 & 7 & -9 & -17 & 17 & 9 & 1 & -7 \\ 7 & -1 & -9 & 17 & 9 & 1 & -7 & 17 \\ 1 & -3 & 3 & -1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[\text{WHT}_{\alpha,\beta}] = \begin{bmatrix} [\text{M}_{\alpha,\beta}^{0,0}] & [\text{M}_{\alpha,\beta}^{0,1}] & [\text{M}_{\alpha,\beta}^{1,0}] & [\text{M}_{\alpha,\beta}^{1,1}] \\ [\text{M}_{\alpha,\beta}^{2,0}] & [\text{M}_{\alpha,\beta}^{2,1}] & [\text{M}_{\alpha,\beta}^{3,0}] & [\text{M}_{\alpha,\beta}^{3,1}] \\ [\text{M}_{\alpha,\beta}^{4,0}] & [\text{M}_{\alpha,\beta}^{4,1}] & [\text{M}_{\alpha,\beta}^{5,0}] & [\text{M}_{\alpha,\beta}^{5,1}] \\ [\text{M}_{\alpha,\beta}^{6,0}] & [\text{M}_{\alpha,\beta}^{6,1}] & [\text{M}_{\alpha,\beta}^{7,0}] & [\text{M}_{\alpha,\beta}^{7,1}] \end{bmatrix}$$

$$[\text{MSH}_{\alpha,\beta}^{k,\ell}] = \begin{bmatrix} [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \\ [\text{MSH}_{\alpha,\beta}^{k,\ell}] \end{bmatrix}$$
By another algorithm, the SHT of order 4, which is also the SWHT of the same order, can be performed with 8 additions and 2 multiplications [3], compared with 10 additions and 2 shifts as given by the above formulas. This order-4 algorithm can be introduced in the recursive definition to trade $2^{n-1}$ additions and $2^n$ shifts for $2^{n-1}$ multiplications in the previously given results.

The SHT has the same number of multiplications and shifts than the SWHT, but has $(n-3)2^{n-4}$ fewer additions and 1 more normalization; therefore the SHT is faster than the SWHT.

OTHER SLANT TRANSFORMS

The two slant transforms considered in this letter are only two members of a large family of slant transforms; this family has been studied in some detail in [7].

CONCLUSIONS

We have defined the SHT and presented some properties of this transform, mainly as compared to the SWHT, which has been successfully considered for image encoding. The SHT is faster and preserves some local properties of the signal and we believe it should be preferred over the SWHT.

REFERENCES


Phase-Plane Analysis of Phase-Locked Loops with Rapidly Varying Phase Error

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Abstract—Nonlinear behavior of phase-locked loops with rapidly varying phase error is examined by using computer phase-plane analysis. The phase variation is modeled by a sinusoidal function. Threshold loop parameters are presented for both sinusoidal and sawtooth phase comparators.

I. INTRODUCTION

The performance of command and telemetry systems, useful in deep-space communications, is frequently affected by the RF phase error which is introduced at the point of reception by means of the carrier tracking loop. In low data-rate communications, the phase error may vary rapidly over the duration of the signaling interval. Causes of this type of behavior in planetary entry are turbulence, dispersion, attenuation, and residual Doppler. The phase variations cannot be tracked by a phase-locked loop of lower bandwidth, while the signal-to-noise ratio (SNR) in this minimum loop bandwidth is too low.

When the ratio of the system data rate to carrier tracking loop bandwidth is less than one, the problem of power allocation between the carrier and the data has been considered by Hayes and Lindsey [1]. In this letter the phase variation is characterized by a sinusoidal input phase, $k \sin (\omega d + \pi/6)$, which models a typical phase variation in communication over turbulent media such as the Venus atmosphere. Conditions for synchronization stability and the acquisition behaviors of the loops in the absence of noise are examined from an

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