Frequency Response Design

We continue with the design of the servomotor from Part 1

Design Problem Statement

A dc motor has the specifications:
\[ J_m = 0.15 \, \text{kgm}^2; b = 1.2 \, \text{Nms/rad}; K_f = 0.6 \, \text{Nm/A}; K_v = 0.6V/s \, \text{rad}; L_a = 0.05 \, \text{H}; R_a = 1.2 \, \Omega \]

to give the motor transfer function:
\[ H(s) = \frac{80}{s(s^2 + 32s + 240)} \]

Design a controller so that the servomotor response to a unit step reference command \( r = 1 \, \text{rad} \), satisfies the following specifications

1. 2% settling time < 1 s
2. Peak overshoot < 10%
3. Maximum steady state error \( |e_\infty|_{\text{max}} = 0.01 \, \text{rad} \) to a constant disturbance torque \( T_d = 0.5 \, \text{Nm} \)

Frequency Response Design

We will use a plot of the magnitude and phase of the open loop frequency response (Bode plot) to help select the controller parameters for the system.

First we will look at the open loop frequency response of the dc motor with the proportional controller we designed with the root locus method:

\[ C(s) = P \text{ with } P = 13.5 \]

Type in the MATLAB workspace

```matlab
>>P=13.5;
>>s=tf('s');
>>C=P;H=80/(s^3+32*s^2+240*s);
>>bode(C*H)
```
MATLAB Control System Toolbox has a command `margin` that gives the stability information from the frequency response:

```
>> margin(C*H)
```

\[ G_m = 17 \text{ dB (at 15.5 rad/sec)}, \quad P_m = 59.1 \text{ deg (at 4.16 rad/sec)} \]
The bode plot shows a crossover frequency of 4.16 rad/s with a phase margin of 59.1°. This margin of stability was able to achieve the overshoot specification and the crossover frequency was also sufficient to achieve the settling time required. However, the steady state error condition could not be met without increasing the proportional gain to 100, driving the system into instability.

We will try a new approach by choosing a proportional-plus-derivative PD controller \( C(s) = P + Ds \) to give a phase margin of 60° at a crossover frequency of 5 rad/s. The derivative action acts as a “damping” effect on the servomotor response, allowing us to increase the proportional gain to reduce the steady state error without destabilizing the system.

First, find the magnitude and phase of the motor frequency response \( H(j\omega) \) at 5 rad/s.

```
>> [magH, phaseH] = bode(H, 5)
```

```
<table>
<thead>
<tr>
<th>magH</th>
<th>phaseH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0597</td>
<td>-126.6561</td>
</tr>
</tbody>
</table>
```

To achieve a phase margin of 60°, the phase lead added by the controller at the crossover frequency must be \(-180 + 60 - (-126.7) = 6.7°\).

**Controller phase:** \( \angle C(j\omega) = \tan^{-1}\left( \frac{D\omega}{P} \right) = 6.7° \implies \frac{D}{P} = 0.0235 \)

The controller magnitude at the crossover frequency must be the reciprocal of the plant magnitude.

**Controller magnitude:** \( |C(j\omega)| = \sqrt{P^2 + (D\omega)^2} = \frac{1}{|H(j\omega)|} \implies P = 16.6; D = 0.39 \)

So the PD controller design is \( C(s) = 16.6 + 0.39s \)

Check the stability margins:

```
>> P = 16.6; D = 0.39;
>> C = P + D*s;
>> margin(C*H)
```
**Build the Simulink model of the dc servomotor control system with PD control:**

![Simulink model of the dc servomotor control system with PD control](image)

**Bode Diagram**

\[ G_m = 27.2 \text{ dB (at 30.8 rad/sec)}, \quad P_m = 60 \text{ deg (at 5 rad/sec)} \]
Run the simulation of the servomotor control system with PD control

As expected the phase margin and crossover frequency ensure that the overshoot and settling time specifications are met and the addition of derivative control leads to a much reduced steady state error \( e_{ss} = r - y_{ss} = 1 - 1.024 = -0.024 \). More “tuning” of the design is needed to reduce the steady state error further.