Introduction to Chaos-Based Communications and Signal Processing[1]

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Abstract—The evolution of the new discipline of nonlinear engineering is taking place along two fronts: the first addressing higher-order effects that have become more important in current designs, while the second more radical activity has focused on the explicit harnessing of nonlinear effects through whole new designs. For the latter case, the most studied nonlinear effect is that of the complex, random-like behavior called "chaos," which is now being applied to such diverse areas as communications, signal processing, fluid mechanics, and physiology. This tutorial survey paper will focus on the application of chaos to the efficiency, reliability, and especially security of information processing and transfer. It will first address the emergence of nonlinear engineering, the basics of dynamical systems, and the technical essentials of chaos. A brief representative tour of some novel applications will then be given, including progress on an effort to realize a microwave chaos-based communications system.

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1. INTRODUCTION

Since its earliest beginnings, the practice of electrical engineering has been dominated by a linear paradigm that has well served the needs for communications signal processing functions. These techniques are well established, mature, and solve a large class of problems, being based on the classical *superposition principle*. This principle states that the response of a given system to a sum of stimuli is given simply by the sum of the responses to each stimulus acting alone. In essence, this viewpoint provided a first-order approximation of a naturally nonlinear world, but this could be done because the engineer is not simply observing and modeling nature like a physicist, but can go beyond this activity to create designs that are intentionally linear and hence obey its simple principles. Because of this, any higher-order nonlinear effects in a design's building blocks resulting from the violation of the superposition principle could be safely ignored. Such effects were often relegated to the catchall terms of "noise" and "distortion," treating them more like an oddity and a nuisance than an inherent and possibly useful feature of nature. This practice, however, could not be done for such important and common signal functions such as frequency generation, frequency synthesis, and power amplification, because these required inherently nonlinear effects to be harnessed. It is these required functions that manifested the first forms of nonlinear engineering, a fledgling new discipline that had already begun to benefit from the pioneering physicists and mathematicians addressing nonlinear systems in the early part of this century.

In view of the above discussion, the development of nonlinear techniques continued to primarily reside within the confines of academia, with physicists and mathematicians as their greatest contributors because of their inherent activity of observation, modeling, and analysis of natural phenomenon. Other than the important examples given above, these new discoveries found little practical application in engineering practice. However, within the last two decades, this situation has experienced a revolution of sorts that stems from three fundamental factors. These factors have synergistically acted together to radically evolve and change the practice of nonlinear engineering—causing a *bifurcation* in the discipline, using the language of nonlinear dynamics.

The first factor is the increased demands for performance in limited bandwidth channels in systems from both commercial and military sectors. As a consequence, nonlinear effects can no longer be ignored in the design of these systems, and must now be addressed by techniques that are not simple extensions of linear theory. A case in point occurs for wide bandwidth communications for advanced military satellites. Here there is often a "bent-pipe" architecture in which the transponder high-power amplifier (HPA) is operated in its nonlinear saturated region for the purposes

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of maximizing power efficiency. This gain in precious efficiency is offset, however, by increased distortion in the modulated signals that pass through the HPA. This distortion is exacerbated by the complexity of the modulations needed to attain high bandwidth efficiencies, since they often contain amplitude variations which elicit these added distortions. This example marks one branch of nonlinear engineering in which nonlinear effects in traditional designs must be characterized and mitigated. In this case, an accurate and formal identification of the HPA must be accomplished before an effective nonlinear compensation strategy can be developed to undue the induced performance-limiting distortion. There is currently a significant amount of activity in this arena fueled by the demands for personal wireless communications.

The second factor involves several seminal discoveries of nonlinear effects that prompted a flurry of research in both elucidating and applying them to communications signal processing, as well as many other disciplines ranging from astronomy to zoology. In essence, a new modeling/analysis language emerged that captured a much larger portion of the complexity of nature. This was an about-face for the previously dominant engineering mindset in that these effects were no longer always undesired, but were instead explicitly sought after for their application potential. This marked the beginning of the second branch of nonlinear engineering in which whole new designs based on the new effects would be sought. These discoveries have primarily occurred in the very active fields of chaos, fractals, and wavelets. Unlike the linear case, this activity is relatively immature and growing tumultuously, and is virtually a wide open frontier of opportunity for new practitioners to make their mark, so to speak. Because the nonlinear methodology necessarily provides a higher-order view of nature in which the superposition principle does not hold, it is typically a very large leap beyond linear thinking, involving much more complex analysis on small classes of problems. This difficulty is, however, offset by the tremendous application potential and import of nonlinear effects.

The final, but in no way the least significant factor has been the rapid development of computer computational power that is imperative for nonlinear study and application. As eluded to earlier, the nonlinear field is characterized by complex problems, most of which do not have closed-form solutions and must be addressed qualitatively and numerically. Coupled with the qualitative arsenal of tools from the discipline of nonlinear dynamics, the computer provided a means to perform nonlinear experiments on the desktop, thereby gaining the valuable insight and knowledge needed to reduce nonlinearity to beneficial practice.

This paper will focus on the first and perhaps most well-known subdiscipline of nonlinear science: chaos, which is many times informally thought of as being synonymous with the nonlinear discipline. In this case the turning-point discovery was that of chaotic synchronization around 1990, allowing the many properties of chaos—a complex, noise-like behavior found in nonlinear dynamical systems—to be applied in a communications context. An essentially whole new era of nonlinear technique development followed in its wake, offering several potential benefits over classical approaches, including, for example, (1) unique privacy and frequency-reuse capabilities even for analog communications (still of military interest); (2) enhanced synchronization performance afforded naturally by the dynamics involved; and (3) several novel signal processing capabilities, several of which are impossible with linear methods. All these benefits contribute to the three fundamental considerations in communications design: efficiency, reliability, and security. Specifically, such designs seek to maximize information density, to be immune to natural and artificial interference, or to ensure that the message sent be received or understood by only an authorized listener.

The purpose of this paper is three-fold. First, it will provide a top-level introduction/survey of the world of chaos and its application impacts on communications signal processing. This is done to make the reader aware of the power and application potential of this subfield of nonlinear science. Second, the paper will seek to provide enough background to evaluate the suitability and relevance of applied chaos to the reader's specialized problem areas. Third, it is hoped that the exposition given here will motivate and provide the resources for the reader to explore the field further, hopefully leading to the growth of the practice of nonlinear engineering. In keeping with this purpose, for example, many of the dynamical systems presented in the paper will provide explicit equations and initial conditions so that the reader can simulate them and see the claimed behavior for themselves.

The paper is organized as follows. First, a quick tour of the language of nonlinear dynamics will be given, including a tour of the technical aspects of chaos and its synchronization, in order to set the stage for the survey of applications that follows. These applications primarily contribute to the security aspect of communications, and will illuminate techniques that could rival and replace traditional approaches. These examples will include: (1) the use of chaos for nonlinear key generation; (2) 1-D and 2-D information encryption; (3) chaotic modulation and demodulation with its natural privacy and *low-probability-of-intercept* (LPI) features, including progress on an internally funded research effort at The Aerospace Corporation that seeks to construct a microwave chaos-based communications system; and (4) other potential novel signal processing applications of chaos that impact the other two aspects of communications design. The paper will conclude with some projections for the future of applied nonlinearity, as well as mention some already well-established uses of fractals and wavelets.

2. Dynamical Systems and Chaos

Fundamentals of Dynamical Systems

The field of dynamics concerns the study of systems whose internal parameters (called *states*) obey a set of temporal rules, essentially encompassing all observable phenomena. This endeavor divides into three subdisciplines, namely:

(1) *applied dynamics*, which concerns the modeling process that transforms actual system observations into an idealized mathematical dynamical system [that is, *state equations* that relate the future states to the past states—usually a set of *difference equations* (DEs), *ordinary differential equations* (ODEs), or *partial differential equations* (PDEs)];

(2) *mathematical dynamics*, which primarily focuses on the qualitative analysis of the model dynamical system; and

(3) *experimental dynamics*, which ranges from controlled laboratory experiments to the numerical simulation of state equations on computers.

The state temporal behavior is either viewed as a traditional *time series* (i.e., a given state parameter versus time) or, more usually, in a *phase space* perspective wherein the *n* system states are plotted against each other in an *n*-dimensional space with time as an implicit parameter (see Figure 1 for the case $n = 2$, adapted from the excellently illustrated text on dynamics by Abraham and Shaw [1]). The latter framework affords a more natural geometrical setting that is reminiscent of a fluid flow, and possesses an arsenal of qualitative analysis tools.

Dynamical systems divide into two major classes: (1) those in which time varies continuously that are usually governed by an ODE or PDE, and (2) those in which time varies discretely that are governed by a DE. Third-order schematic illustrations of this division are shown in Figure 2. In part (a) of the figure, the *vector field* $\mathbf{F}$, which is the result of the modeling process, provides the direction (tangent vector) by which the system, at the point $\mathbf{x}$ and time t, moves forward in time, tracing out the *trajectory* that is the behavior of the dynamical system. This dynamical system is either a state equation of the form

$$\frac{d\mathbf{x}}{dt} =: \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}), \tag{1a}$$

called an *autonomous* or *unforced* ODE, or one of the form

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, t), \tag{1b}$$

called a *nonautonomous* or *forced* ODE. In the case of a PDE, there is no such simple tangential vector field interpretation since the solution is a quantity (or quantities) that varies with the spatial variables and time, and not a vector that varies with time[1]. In a similar manner, part (b)

[1]For simplicity, we will henceforth assume that a continuous dynamical system is always governed by an ODE.

(a)

(b)

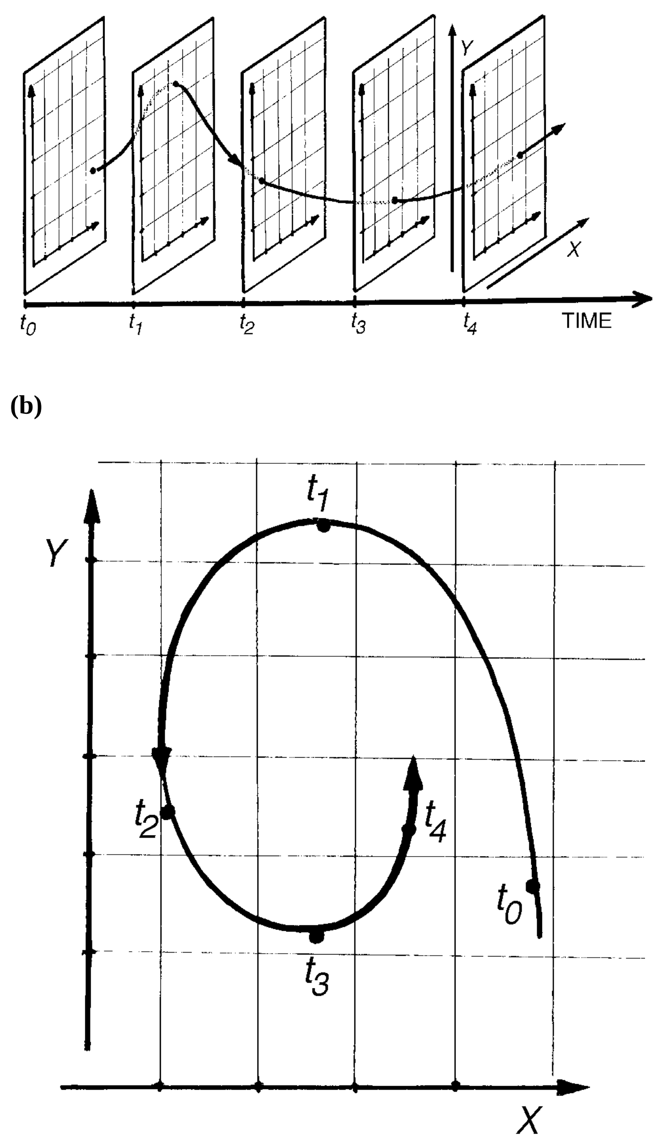


Figure 1: *Two different perspectives for representing the temporal evolution of dynamical state variables.* **(a)** *Time series for a two-dimensional dynamical system.* **(b)** *Planar phase space orbit obtained by projecting out the time parameter in* **(a)**.

of the figure illustrates a *state transition map* $\mathbf{\Phi}$ that dictates where the system, at the point $\mathbf{x}$ and time t_2, moves at the next allowed time t_3, tracing out the *orbit* that is the behavior of the dynamical system. In this case this system is a DE of the following two state equation forms:

$$\mathbf{x}_{n+1} = \mathbf{\Phi}(\mathbf{x}_n) \quad \text{(autonomous)} \tag{2a}$$
$$\mathbf{x}_{n+1} = \mathbf{\Phi}(\mathbf{x}_n, t_n) \quad \text{(nonautonomous)} \tag{2b}$$

The former class of dynamical systems would be associated with analog circuits, while the latter class would be associated with digital circuits, although the value of the state variables is not normally quantized in a discrete dynamical system.

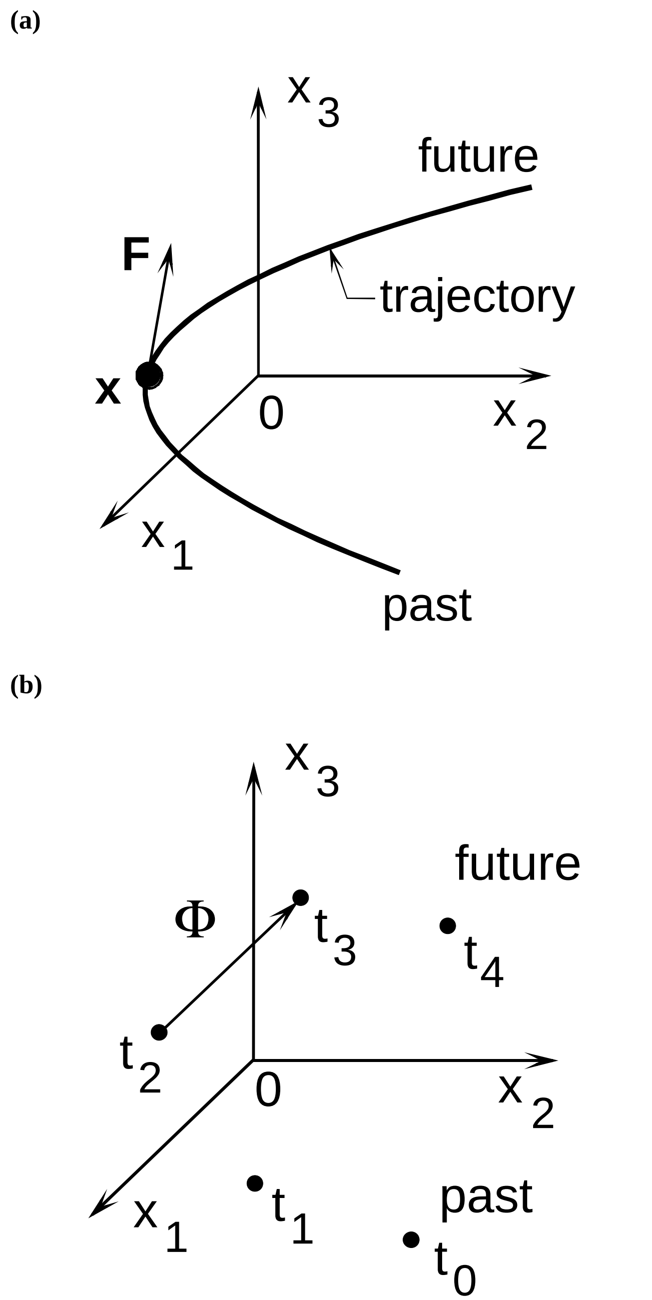


Figure 2: *Third-order representative examples of the two basic classes of dynamical systems.* **(a)** *Continuous system governed by a vector field* $\mathbf{F}$. **(b)** *Discrete system governed by a state transition map* $\mathbf{\Phi}$.

A dynamical system is said to be *linear* or *nonlinear* depending on whether the *superposition rule* holds for its governing vector field or state transition map. In particular, this would mean whether the following relation holds or not (letting $\mathbf{f}$ represent either of the two functions):

$$\mathbf{f}(\alpha\mathbf{v} + \beta\mathbf{w}) \stackrel{?}{=} \alpha\mathbf{f}(\mathbf{v}) + \beta\mathbf{f}(\mathbf{w}), \qquad (3)$$

for all constants α, β and vectors $\mathbf{v}$, $\mathbf{w}$. Other classifications of dynamical systems include: (1) *dissipative* or *lossy*, corresponding to most physical systems (e.g., forced pendulum with friction); and (2) *conservative* or *lossless*, corresponding to classical Hamiltonian and quantum systems (e.g., planetary motion). The solutions to these systems are also often divided into segments designated as *transient* or *steady-state*, indicating the short-term and asymptotic portion of the entire temporal solution. Examples of the above classes of dynamical systems, and other concepts of nonlinear dynamics will be given and introduced throughout the paper. Besides [1], a good glossary of the language of dynamics can be found in [2], as well as the technical nonlinear dynamics texts [3]–[9].

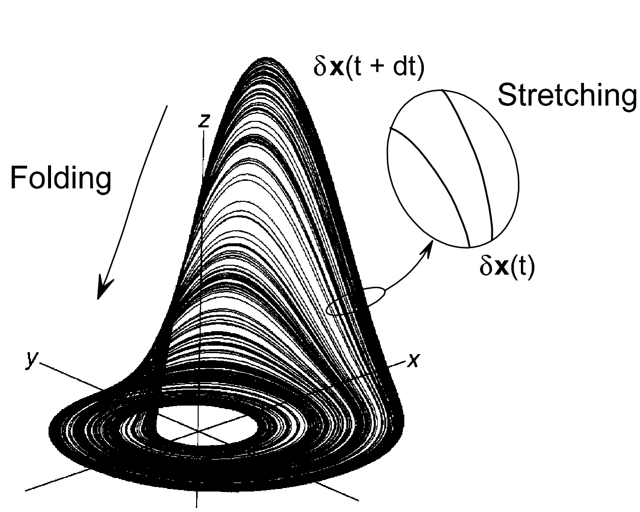
Technical Essentials of Chaos

One of the most well-known and potentially useful nonlinear dynamical effects is the bounded, random-like behavior called *chaos*—in essence, “deterministic noise.” Chaos has been found to occur in a whole myriad of dynamical systems modeling phenomena from astronomy to zoology, and in frequency ranges from baseband to optical. It has even found its way into the popular press and media, with several general audience books and articles to its credit (e.g. [3, 7, 10, 11, 12]). Likewise there have been many excellent technical texts, general articles, and special journal issues covering this subject, such as [4, 5, 6, 9] and [13]–[19]. This phenomena; its closely related *fractal* cousin—which is a complicated limiting set primarily produced by *affine* maps (that is, linear maps with a constant offset) on the complex plane (see the exceptional comprehensive text [13]); and the mathematical tool called *wavelets* which has experienced an explosive growth over the last decade (see [20], for example); have been put forth as a new paradigm for the understanding and modeling of the world around us. This stems from their common underlying principle of *self-similarity at different scales* that appears to be a ubiquitous property of nature.

Beneath the superficial notions of chaos that appear in the general media, there is a real technical side to chaos that is quite deep both mathematically and philosophically. With respect to its technical aspects, there are three basic dynamical properties that collectively characterize chaotic behavior: (1) an essentially continuous and possibly banded frequency spectrum that resembles random noise; (2) *sensitivity to initial conditions*, that is, nearby orbits diverge very rapidly; and (3) an *ergodicity and mixing* of the dynamical orbits, which in essence implies the wholesale visit of the entire phase space by the chaotic behavior, and a loss of information. This is a practical definition of chaos that has a corresponding set of several alternative technical ones, none of which has become the official definition (see [4, 9] for some of these definitions).

These traits are suggested and illustrated in Figures 3 and 4 which provide several fundamental and well-known examples of chaotic dynamical systems. These traits will also arise in our presentation below concerning chaos-based communications. First, Figure 3 provides illustrations

(a)



(b)

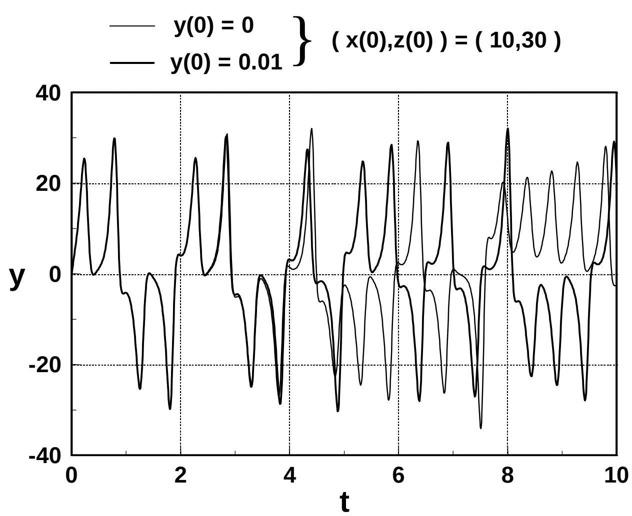


Figure 3: *Some characteristic features of chaotic behavior.* **(a)** *Chaotic strange attractor from the third-order Rössler system.* **(b)** *Illustration of sensitivity to initial conditions in the third-order Lorenz system.*

of two such continuous autonomous systems. The top portrait depicts what is called a *strange attractor*—a primary manifestation of chaotic behavior—in a prototypical third-order dynamical system (known as the Rössler system [21]). This steady-state behavior is called an attractor because trajectories begun outside of the set are attracted and influenced by it, and it is called strange because the topology of the set is extremely complex. The complexity and boundedness of the attractor stems from the characteristic stretch-and-fold operation illustrated in the portrait, and leads to its fractional dimension (i.e., space-filling nature). It is the repetition of such a simple operation that seems to be at the heart of much of the complexity found in nature, as the field of fractals especially shows. The strange attractor shown was obtained from the following equations:

$$\dot{x} = -(y+z),\ \dot{y} = x + \frac{1}{5}y,\ \dot{z} = \frac{1}{5} + z(x-\mu), \tag{4}$$

with the parameter μ set equal to 4 and the initial state $(x(0), y(0), z(0))$ set to $(-1, 0, 0)$.

Figure 3(b) illustrates the sensitivity to initial conditions for another prototypical third-order chaotic system (called the Lorenz system [22]; also reprinted in [14]). The strange attractor for this system was perhaps the first discovered from a computer model of a natural phenomenon, in particular, thermal convection in the atmosphere. In this case, the equations represented the first three primary modes of the actual PDE that is needed to characterize the system. Observe from the figure how only a slight change in the initial *y*-coordinate value y_0 leads quickly to very different orbital futures, sometimes referred to as the "butterfly effect." It is this effect that leads to a rapid loss of information and expansion of any errors, thereby making long-term weather prediction, for example, specious at best. This expansion rate is quantitatively captured in what are called *Lyapunov exponents*, with positive exponents providing a very strong indication of chaotic behavior (see [5], for example). In this case, the equations used were:

$$\left.\begin{array}{rcl} \dot{x} & = & \sigma\,(y-x) \\ \dot{y} & = & R\,x - y - x\,z \\ \dot{z} & = & -B\,z + x\,y \end{array}\right\} \tag{5}$$

where the physical parameters σ, B, and R take on the values 10, 8/3, and 28, respectively, and the initial conditions are as shown in the figure. A clear indication of the immense complexity of even this simple system is seen from the entire book devoted to it by Sparrow [23].

Figure 4 provides an autonomous 1-D prototypical example from the discrete dynamical system class known as the *logistics map* used to model population dynamics (see [24] or its reprinted form in [14]). This example also serves to illustrate the fundamental concept of *bifurcation* in nonlinear dynamical systems, that is, the abrupt changing of the qualitative behavior of a system caused by the variation of some set of parameters. The investigation into the engineering implications and applications of this unique and rich phenomena is essentially totally unexplored. In this case the dynamical system is given by

$$x_{n+1} = \mu\, x_n\,(1 - x_n) =: f(x_n), \tag{6}$$

where the parameter μ varies in the interval $[0, 4]$ so that $f(x_n)$ maps the unit interval into the unit interval. The richness of the behavior of this simple system is again attested to by the excellent and detailed exposition in [25]. Part (a) of the figure shows normal orderly behavior found with μ set below the special value of 3. Note how the orbit eventually converges towards the *stable fixed point* x_e which satisfies the condition $f(x_e) = x_e$. In part (b),

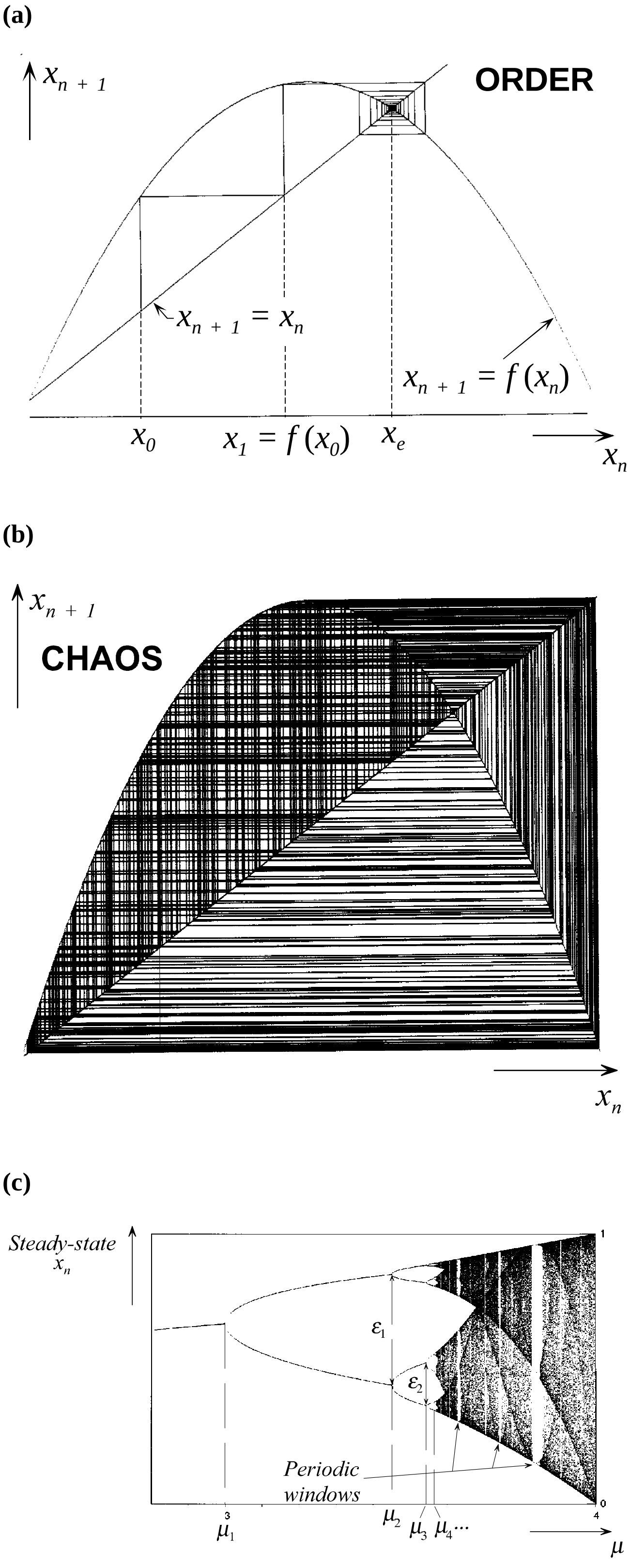


Figure 4: *One-dimensional logistic map example of a discrete dynamical system.* **(a)** *Simple asymptotic behavior found with the bifurcation parameter μ set below 3.* **(b)** *Complex chaotic behavior observed with $\mu = 4$.* **(c)** *Bifurcation diagram illustrating the period-doubling route to chaos and periodic windows.*

a much different complex picture arises with $\mu = 4$ that would be considered chaotic. In this case, the previous fixed point x_e is now *unstable*, so that orbits keep getting repelled from it and return to it to be repelled again, and so on. Finally, part (c) of the figure serves to illustrate the very interesting *period-doubling* route between the ordered and chaotic behavior. This so-called *bifurcation diagram* is also a very common perspective in nonlinear dynamics, being in this case a plot of the steady-state behavior of Eq. (6) with respect to the *bifurcation parameter* μ. Observe how the first bifurcation occurs at the value of 3, followed by further doublings at shorter and shorter intervals of μ until the period goes to infinity at $\mu_\infty = 3.449490\ldots$, signifying chaos. Also observe the so-called *periodic windows* interspersed beyond μ_∞ in which the behavior returns to a normal periodic one, quickly followed again by bifurcations to an infinite period. This is one of but several routes to chaos (see the survey in [26]), each of which are entire studies in and of themselves. A fundamental discovery about the period-doubling route was its *universality* in nonlinear systems and the almost π-like constants associated with its structure (see [27]). Specifically, Feigenbaum showed that for the quantities shown in the figure,

$$\lim_{i \to \infty} \frac{\mu_{i+1} - \mu_i}{\mu_{i+2} - \mu_{i+1}} =: \sigma = 4.6692016091\ldots \tag{7a}$$

for the period-doubling bifurcation points and

$$\lim_{i \to \infty} \frac{\epsilon_i}{\epsilon_{i+1}} =: \alpha = 2.5029078750\ldots \tag{7b}$$

for the branch splittings ratios. This profound property is intimately related to the self-similarity property discussed earlier. This property is especially evident in the 2-D mapping of Hénon [28] which is yet another fundamental example of a dynamical system with a strange attractor.

There are several important observations and classifications of chaos that should be noted for completeness. First, chaos can only occur in *nonlinear* continuous systems described by ODEs of third order or higher and PDEs of any order, and in nonlinear discrete systems described by DEs of any order. For both ODEs and DEs, a forcing function that is explicitly a function of time provides an additional state variable. Thus a forced second-order system can be thought of as a third-order system and hence could be chaotic in both the continuous and discrete cases. Similar to the case of orbits in general, chaotic behavior can be classified into several types. *Transient chaos* usually occurs in periodically forced systems such as analog phase-locked loops, giving way asymptotically to orderly behavior. *Intermittent chaos* or *intermittency* as it is commonly called, usually occurs when a mapping exhibits a form of criticality that leads to behavior with chaotic episodes interspersed with regular behavior at random periods. This form of chaos is often chosen as a model for the generation of the much-studied $1/f$ noise phenomena that arises commonly in nature (see [5], for example). In dissipative systems, steady-state chaotic

behavior is called a strange attractor as discussed above, while in conservative systems it is termed *strong* or *weak stochasticity*, depending on whether the behavior is globally or locally resident in the phase space. It should be noted that in principle, a chaotic system can include an infinite number of both periodic orbits of any period and nonperiodic orbits. These unique properties have led to several of the applications to be presented below.

Basics of Chaotic Synchronization

The classical synchronization (or *entrainment*) of periodic oscillators has been known since at least the seventeenth century when Christiaan Huyghens observed the coupled form of the phenomenon in clocks on a wall. The driven or injection form of synchronization was discovered later when it was observed that a small periodic forcing signal could cause the large natural resonance (or a harmonic or subharmonic thereof) of a system to lock to it. What no one could expect was that a similar phenomenon could be had with chaotic signals, especially given its distinctive bounded instability character. Nevertheless, the discovery of the driven form of *chaotic synchronization* was announced in 1990 [29], marking a turning point in the investigation of chaos for applications, for it allowed chaos to be modulated and demodulated like a generalized carrier.

Four basic chaotic synchronization techniques have already been discovered, the first two of which are shown in Figures 5 and 6 and will be discussed in more detail here:

(1) *Method I:* An autonomous system unidirectionally driving a stable subsystem, often termed the *master-slave* approach [29].

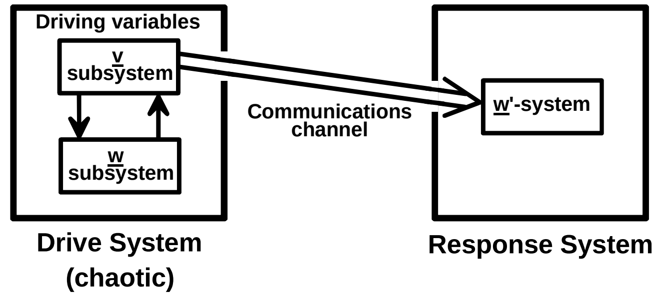
(2) *Method II:* A nonautonomous system unidirectionally driving a stable identical nonautonomous system [30].

(3) *Method III:* Adaptive control approach which has many variants with [31] as one of its earliest examples. Other selected examples here include [32]–[36].

(4) *Method IV:* Bidirectionally coupled identical systems [37, 38].

Only the first three of these forms of chaotic synchronization would be suitable for standard communications purposes. In addition, of course, it is preferable that the linking signal between the component systems also be of the scalar variety. Because of the newness of these discoveries, many studies are still needed to address important engineering and operational issues, and to compare findings with traditional synchronization approaches. Some of these issues and any related representative investigations include: (a) choosing/designing the chaotic systems that will synchronize [39]–[43]; (b) the region and rate of synchronization which in this case is dynamically natural and automatic; (c) immunity of synchronization to linking signal perturbations (interference, filtering, and random noise) [30, 44]; (d) relation of synchronization performance characteristics to system parameters; and (e) the inference of the chaotic system used in the link based on observation of the linking signal (security issue) [45]. The uniqueness of this generalized form of synchronization lies in its dynamical naturalness, simplicity of implementation, and security properties that are not commonly shared by its classical periodic counterpart.

(a)



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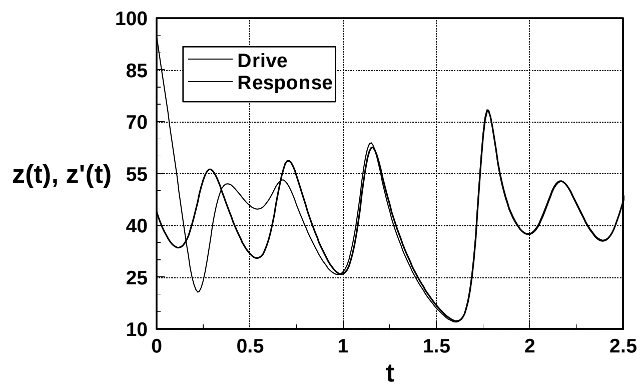


Figure 5: *The master-slave form of chaotic synchronization.* **(a)** *Schematic illustration.* **(b)** *Computer simulation of a Lorenz system example using the state variable x as the drive.*

The master-slave form of chaotic synchronization is the primary and earliest approach discovered that made chaos-based communications possible. Figure 5(a) provides a schematic illustration of the method, wherein a chaotic system is divided into two subsystems, one of which is replicated remotely (called the *response system*), while the other is used to drive the response system. The decomposition of the drive system is given explicitly by

$$\left.\begin{array}{c}\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \\ {[\mathbf{x} = (\mathbf{v},\mathbf{w})]}\end{array}\right\} \underset{\text{decomp.}}{\longrightarrow} \left\{\begin{array}{l}\dot{\mathbf{v}} = \mathbf{g}(\mathbf{v},\mathbf{w}) \\ \dot{\mathbf{w}} = \mathbf{h}(\mathbf{v},\mathbf{w})\end{array}\right. \tag{8}$$

and the replicated $\mathbf{w}'$-response system is governed by

$$\dot{\mathbf{w}}' = \mathbf{h}(\mathbf{v},\mathbf{w}'), \tag{9}$$

where the vector $\mathbf{v}$ represents the drive variables. In its simplest continuous instance, the entire drive system would be third-order, while the $\mathbf{w}$-, $\mathbf{w}'$-subsystem would be second-order, meaning that only a scalar variable v is transmitted across the communications channel linking the drive and response systems. Appropriate rigorous conditions insure the chaotic synchronization of the $\mathbf{w}$- and $\mathbf{w}'$-subsystems, and they allow for real-world parameter mismatches. An explicit example of this synchronization is shown in Figure 5(b) for the previously described Lorenz system. In this example, the linking signal was provided by the state variable $v = x$, with the drive system governed by Eq. (5)—where $\sigma = 16$, $B = 4$, and $R = 45.92$ in order to elicit chaotic behavior—and the response system governed by

$$\left.\begin{array}{rcl} \dot{y}' & = & R\,x - y' - x\,z' \\ \dot{z}' & = & -B\,z' + x\,y' \end{array}\right\} \qquad (10)$$

Observe how the chaotic $z(t)$ and $z'(t)$ state variables quickly converge despite their widely separated initial conditions. This indicates the robustness of the synchronization phenomena for this baseband circuit, whose convergence rate will also scale up with the frequency of operation. A similar finding occurs for $y(t)$ and $y'(t)$. One of the earliest experimental demonstrations of this phenomena used Chua's circuit [46], another prototypical chaotic system that is the simplest and most-studied chaotic circuit in electrical engineering [47]. A related form of synchronization is discussed for discrete systems in [48].

In parallel with the previous form, the nonautonomous variety of chaotic synchronization is schematically represented in Figure 6(a), and experimentally shown in Figure 6(b) for a modified version of the classical Duffing oscillator to be discussed below [30]. In this less mature scheme, the frequency and phase of the response system's forcing function must be adjusted with an adaptive technique in order to line up with the forcing function for the drive system. The immaturity here comes in with how to feed the adjustment to insure synchronization, since one essentially has to determine a generalization of the phase and frequency error between two chaotic signals. In this way, the system becomes a *chaos-locked loop*, generalizing the traditional phase-locked loop that operates on sinusoidal signals. Some work along these lines can be found in [49]. A simple error detector scheme was devised for the experiment shown in part (b) of the figure, which was also found to be extremely robust with respect to interference added to the link, both in the form of white noise and another uncorrelated chaotic signal of *twice* the amplitude of the driving signal x. Such findings motivated our own work in the area of chaos-based communications to be discussed below. Another demonstration of this phenomenon using a forced version of Chua's circuit is given in [50].

The following sections will briefly enumerate representative applications that have been demonstrated/proposed for chaos, offering a glimpse of the power and potential of this promising area of applied nonlinearity. A representative selection of such applications is covered in the review articles and conference proceedings in [51]–[54].

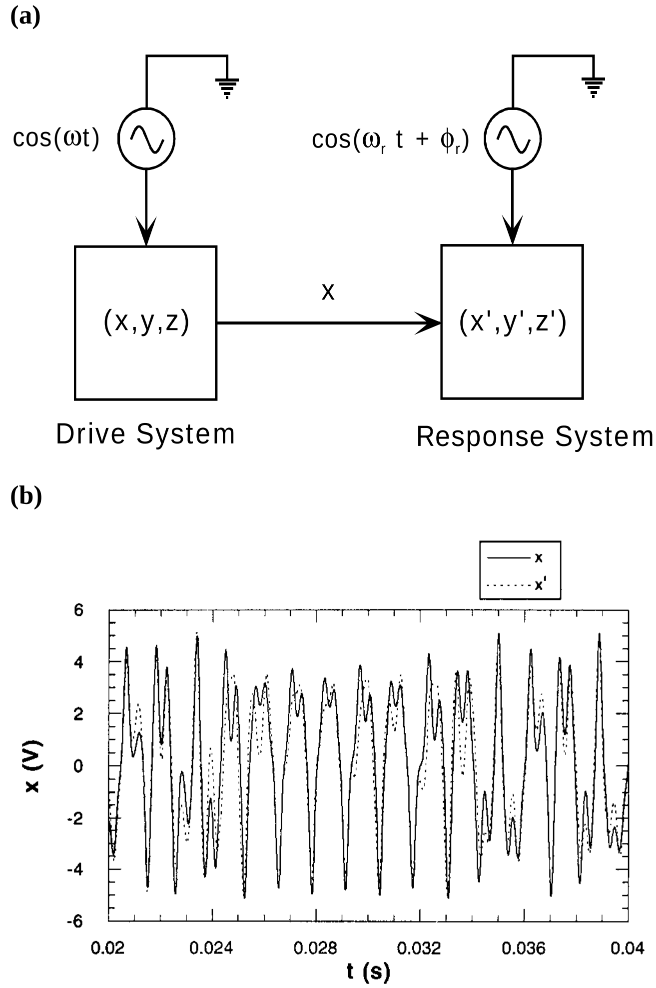


Figure 6: *The nonautonomous form of chaotic synchronization.* **(a)** *Schematic illustration for a third-order, scalar-driven example.* **(b)** *Modified one-well Duffing system example using the state variable x as the drive (from [30]).*

3. Pseudo-Random Sequence Generation

Probably one of the earliest applications of chaos came from the observation of its natural pseudorandomness, either as a sampled form of continuous chaos, or straight from the appropriate nonlinear map. Employing the latter, several chaotic key generators were proposed to replace traditional linear and nonlinear feedback shift registers—many of which were known to be susceptible to cryptanalysis schemes—for use in standard digital cryptographic and spread-spectrum systems. A good overview of these and other schemes during this period can be found in [55]. Perhaps the first of the chaos-based generators was the one based on a generalized logistic map reported by Matthews [56] and given explicitly

by

$$x_{n+1} = g(x_n) = (\beta + 1)\left(1 + \frac{1}{\beta}\right)^{\beta} x_n\,(1 - x_n)^{\beta}, \quad (11)$$

where $1 \le \beta \le 4$, $0 < x < 1$. The ciphering procedure consisted of specifying β and x_0 to D digits, calculating random iterates to $D + 2$ digits, setting n^{th} key value k_n to $(D-1)^{\text{st}}$ and D^{th} digit of iterates, and finally adding the resulting keystream to plaintex. Although this and other early versions of this approach were susceptible to short-cycling problems (because of the existence of periodic orbits of all periods within strange attractors), improved modifications were shown to rival classical feedback shift registers in passing standard randomness tests [57, 58]. For example, the improved generator of Mitchell [57] used a monotonic iterative scheme of the form

$$x_{n+1} = \sum_{i=1}^{k} a_i (d_i\, x_n^{g_i} + c_i)^{b_i}, \quad (12)$$

where $a_i, b_i > 0$, $b_1 > 1$, and c_i, d_i, g_i are appropriately chosen. Here the key sequence k_n is given by the $(D-1)^{\text{st}}$ and D^{th} digits of x_n. Because x_n monotonically increases with n, there can be no cycling problems. Although x_n is not chaotic here the key sequence k_n is: being bounded, nonmonotonic, and noncyclic. The combination of the outputs of several of these enhanced generators can lead to ciphering procedures that rival the classical *Data Encryption Standard*. More recent results and their applications concerning chaos-based generators can be found in [59]–[62]. Further significant enhancement of these generators could be accomplished with what are called *control chaos* techniques (see [63], for example), in which dynamical systems are steered to a desired orbit via small parameter changes. An example of such a desired orbit would be a periodic one having a very long period that is known to exist in strange attractors.

4. MAPPING-BASED ENCRYPTION

In a similar vein, chaotic and quasi-chaotic nonlinear maps (both 1D and 2D) have been used as the basis for data and image encryption. The idea here is that simple nonlinear maps can give rise to very complicated behavior in only a few iterations; and if the process is reversible, then encryption and decryption can be accomplished efficiently and with a good degree of security. This security is embedded in the nature of the map and its parameters, the former of which must exactly match between the sender and receiver, while the latter can only allow for very small discrepancies that are practically attainable, however. The following two examples serve to illustrate these approaches, the first involving a single data stream, while the second entails the encryption of images:

(1) Figure 7 illustrates a piecewise-linear (PWL) *tent map* F restricted to the unit interval and parameterized by $\alpha \in$

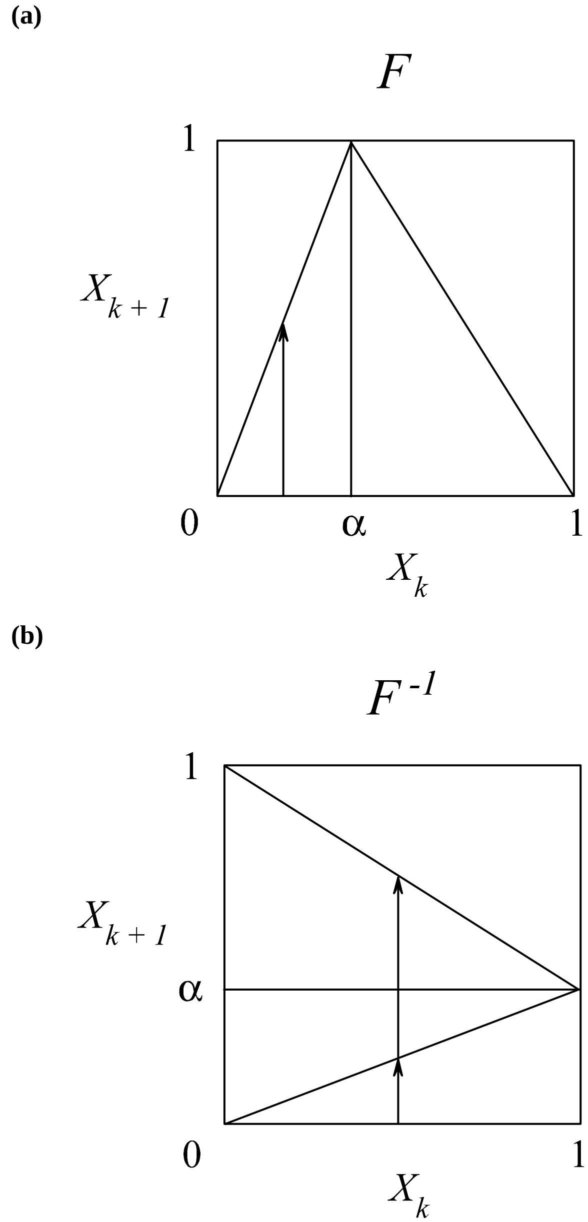


Figure 7: *One-dimensional tent map encrpyter (from [64]).* **(a)** *Forward map.* **(b)** *Bi-valued inverse map.*

$(0, 1)$, as proposed by Habutsu [64]. Note that the inverse map F^{-1} has two branches which must be chosen in the encryption procedure. Explicitly, the map and its inverse is given by

$$F:\ X_{k+1} \;=\; \begin{cases} \dfrac{X_k}{\alpha}, & 0 \le X_k \le \alpha, \\[2ex] \dfrac{X_k - 1}{\alpha - 1}, & \alpha < X_k \le 1 \end{cases} \quad (13a)$$

$$F^{-1}:\ X_{k-1} \;=\; \alpha X_k \ \text{ or } \ (\alpha - 1)X_k + 1 \quad (13b)$$

Here α serves as the secret key, and the encryption procedure is as follows: choose the initial condition x_0 to be equal to the plaintext value $p \in (0, 1)$, where $p \neq \alpha$; and

calculate the ciphertext using $c = F^{-n}(p)$, choosing either branch of F^{-1} for each iterate. The decryption procedure is simply $p = F^n(c)$. Typical parameter choices here are $\alpha \in (0.4, 0.6)$; α and p: 64 bits, $n \approx 75$, c: 44 bits. A more sophisticated scheme to perform this type of encryption involved the use of the classical two-dimensional Hénon map [65] discussed earlier.

(2) Figure 8 illustrates an image encryption procedure involving a quasi-chaotic quantized set of two tent maps that act on the intensity $I(x, y)$ of the given image. This work was reported on in [66]–[68] which illuminated the interesting effects of nonlinear pictorial feedback systems (analog and digital). To be more precise, the encoding and decoding algorithms for this scheme were given by

$$I_o(x,y) = NL_2^{(k)}\{NL_1^{(m)}[I_i(x,y)] + Rd(x,y)\} \quad \text{(14a)}$$
$$I_r(x,y) = NL_1^{(Q_1-m)}[NL_2^{(Q_2-k)}(I_o) - R_d] \quad \text{(14b)}$$

where $I_i(x, y)$ is the quantized intensity in the input image; $NL_j,\ j = 1, 2$ are the two quantized nonlinear operators depicted in parts (a) and (b) of Figure 8; $k,\ m$ are the number of cycles chosen to iterated the operators; $I_o(x, y)$ is the quantized intensity for the scrambled image; $Rd(x, y)$ is a frozen noise pattern added after the inner iteration of the algorithm; $I_r(x, y)$ is the quantized intensity of the recovered image; and $Q_j,\ j = 1, 2$ is the period of the quantized operator NL_i. The frozen noise pattern is added in order to remove contour lines of the objects in the images caused by roundoff of the intensities that takes place here. Observer that NL_2 is a "noisy" nonlinearity, where the size of the dots indicates the relative probability of mapping the j^{th} value of the intensity into the i^{th} value. The security of this scheme comes from several components, namely, the nature of the two nonlinearities, the number of times they are each iterated, the number of quantization levels, and the frozen noise pattern chosen. Part (c) of the figure illustrates an example of the use of this scheme. A more recent sophisticated scheme based on invertible two-dimensional maps is reported in [69].

5. COMMUNICATIONS WITH CHAOS

Survey of Techniques and Baseband Demonstrations

A whole series of baseband communication links have been demonstrated both by simulation and experiment. These simple prototypes were based on the various forms of chaotic synchronization and modulation schemes that have been developed (the latter ranging from simple additive masking to indirect parameter modulation that could offer enhanced message privacy/security) [70]–[79]. The rationale for these investigations is that this new approach to communications harbors several potential advantages over current techniques. Some of these features include: (1) digital and analog implementations that synchronize more rapidly, robustly, and simply because of their natural dynamical properties; (2) unique analog communications capabilities (such as privacy,

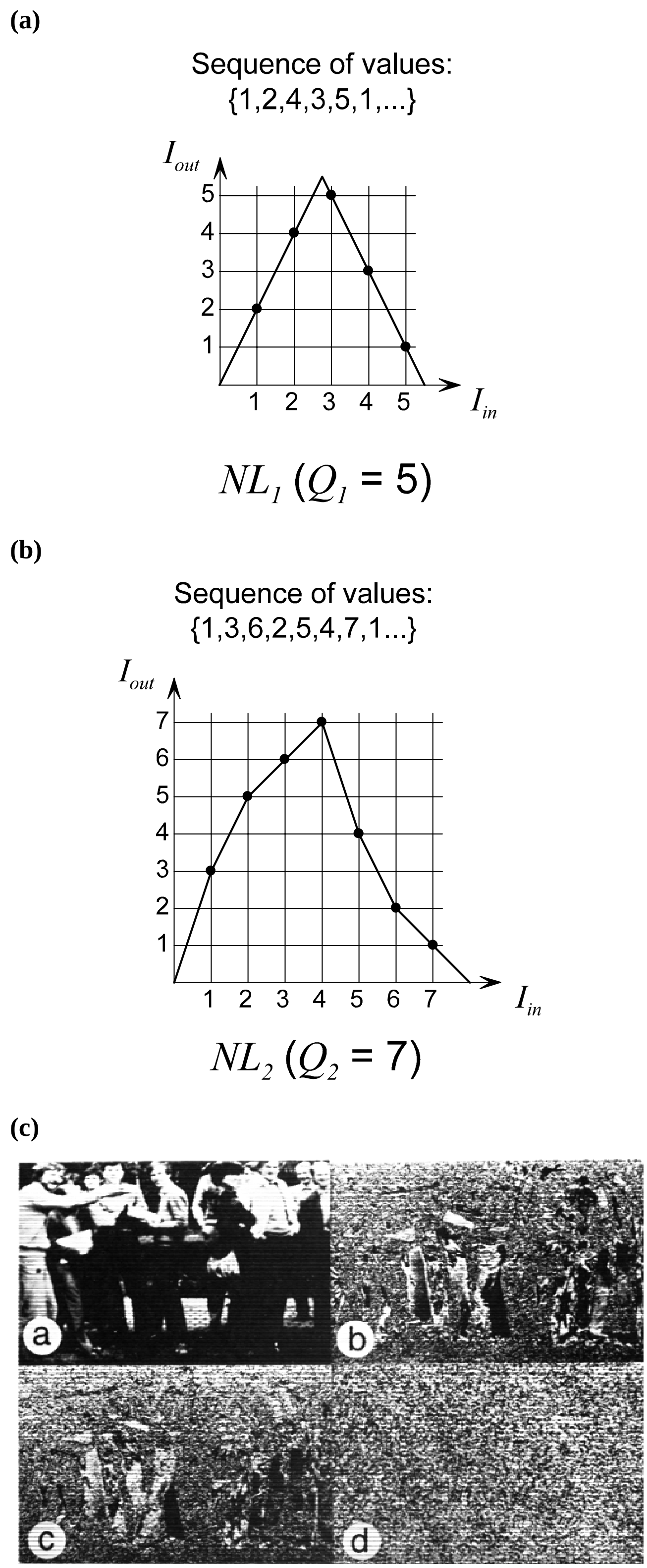


Figure 8: *Image encryption based on quantized intensity tent maps (from [68]).* **(a)** *Inner nonlinear intensity map* NL_1. **(b)** *Outer nonlinear intensity map* NL_2. **(c)** *Example of image encryption with the following legend: a — initial and "restorable image, b —* m *times through* NL_1*, c — frozen noise pattern added, d — final scrambled image after* k *cycles of* NL_2.

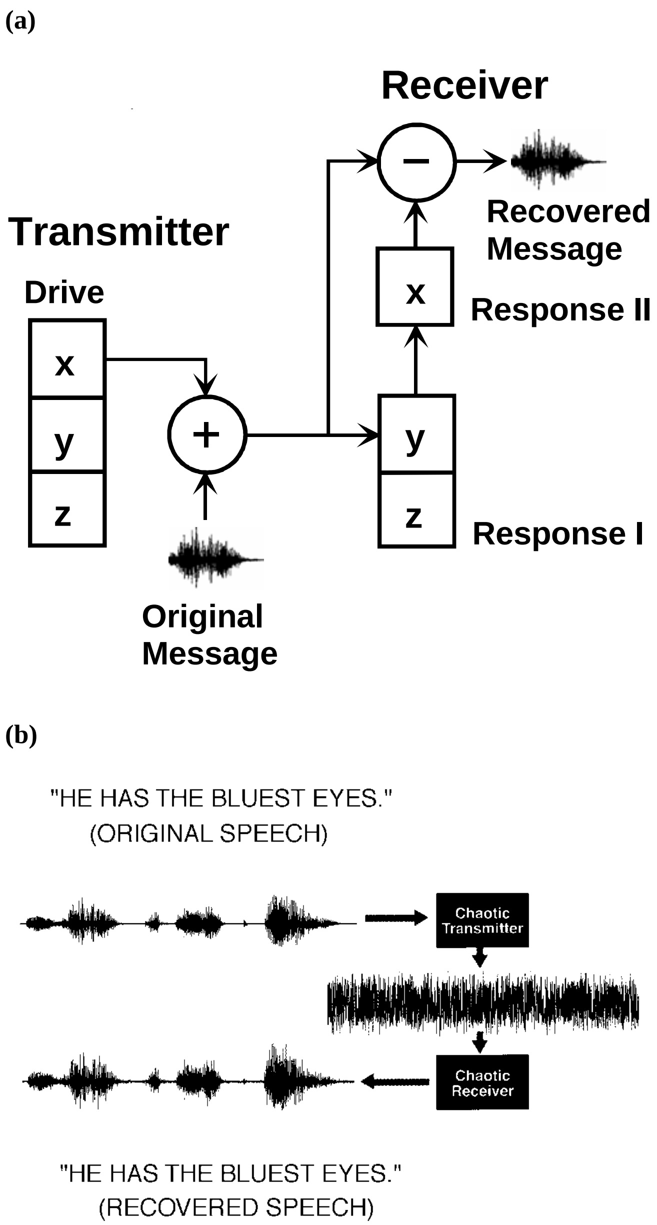


Figure 9: *Example of chaotic masking modulation, one of several means for chaotic communications [80].* **(a)** *System configuration.* **(b)** *Experimental results for speech.*

low probability of intercept, and frequency reuse) which are still of interest to the military sector; and (3) other unique signaling functions not possible with digital techniques (such as indirect chaotic modulation, chaotic signal constellations, and noise reduction that are discussed below). At this stage the efforts can be primarily characterized as explorational—just seeing what can be done and what really are the advantages (if any) of using these novel techniques for classical communication purposes.

Figure 9 illustrates one of first reported forms of chaos-based communications [80], that used a cascaded form of master-slave synchronization (see [81] for details) and additive chaotic modulation. The cascading is needed

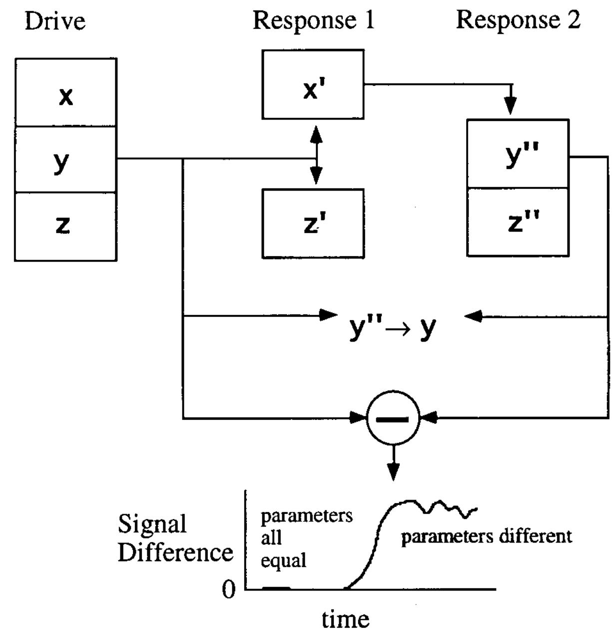


Figure 10: *Cascaded architecture for realizing parametric modulation (from [81]).*

in order to locally regenerate the chaotic carrier. This system was found to be quite resilient to noise/interference added to the linking channel, as is needed for a pragmatic communications system. Part (a) of the figure shows the transmitter/receiver configuration that is based on the previously mentioned Lorenz system. In this case, the chaotic carrier x is modulated by adding a voice message at a much lower level, and is recoverable since the chaotic carrier is locally coherently regenerated in the receiver. In part (b), actual experimental results are shown for the chaotic communication system in (a) using baseband speech as the message. Note how the message is buried in the "noise" when viewed in the communications channel, indicating how this approach can possibly provide for LPI and private transmissions. One must be careful about making such claims, as was often done early on in the development of chaos-based communications, since, for example, Short [45] showed that the additive modulation scheme depicted here is very easily deciphered using what are called *de-embedding* techniques. These techniques seek to determine the dynamical system behind some observable and themselves have yet to be applied to traditional digital encryption schemes (since they can be thought of as sophisticated mappings of the plaintext).

Another more sophisticated example of chaotic modulation that cannot be imitated by traditional approaches and is much more secure is illustrated in Figures 10 and 11. In this case, the message modulates a chosen circuit parameter in the system, which in turn influences the state variables of the system in a very complex manner. Because the state variables, or combinations thereof, are the signals sent across the communications channel, the manner in

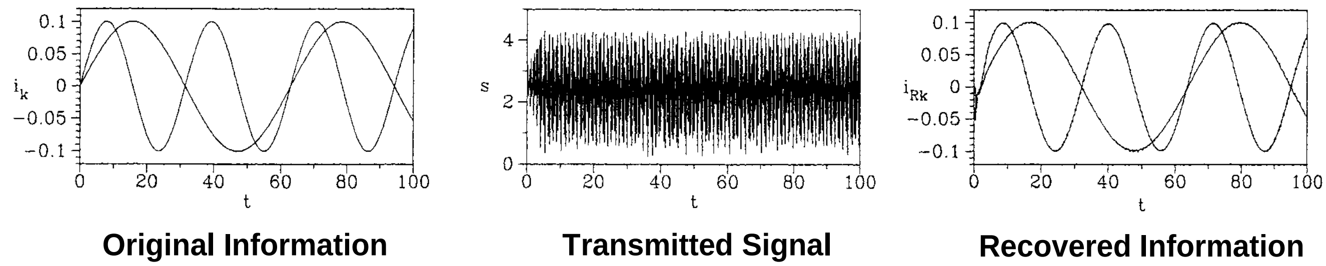


Figure 11: *Simulation results for multiple parameter modulation as reported in [78].*

which the original message is embedded in this signal is extremely complex, not simply a modulation of the phase and/or amplitude of some sinusoidal carrier. This form of chaotic modulation really brings home the point of security in the carrier without even considering the encryption of the message before being modulated.

Figure 10 sketches how such parameter modulation takes place. It requires a cascaded structure and some adaptive control techniques. Essentially, the parameter in the receiver that was modulated in the transmitter must be controlled in order to preserve the chaotic synchronization. The control signal used for this then turns out to be the demodulated information. Figure 11 shows simulation results for an extension of this basic scheme that involves the modulation of several parameters with independent messages, forming a chaos-based multiplexing of information onto one channel. This particular example employed Chua's oscillator in which the value of one of the capacitors and the inductor were modulated with independent sinusoids.

As far as real-world applications, these designs can be directly used for baseband communications, whereas for RF communications, these schemes must be combined with traditional carriers and modulation/demodulation techniques. In both cases, the bandwidth of the information is limited to tens of KHz, while in the latter case, an additional loss of LPI capability must be suffered. Similar to synchronization discussed above, there are several important engineering issues that must now be addressed before operational application can be considered for these new communications approaches. In fact, the issues already mentioned above carry over to the case of modulation and demodulation as well. An excellent overview of these concerns is given in the two-part paper [82, 83].

Progress in Microwave Chaos-Based Communications

Motivated by these current limitations for chaotic communications, the authors have been conducting an internally funded project at The Aerospace Corporation that seeks to investigate and realize high-frequency chaotic communications systems. The first stage of this project has been the development of a high-frequency chaotic oscillator. This has

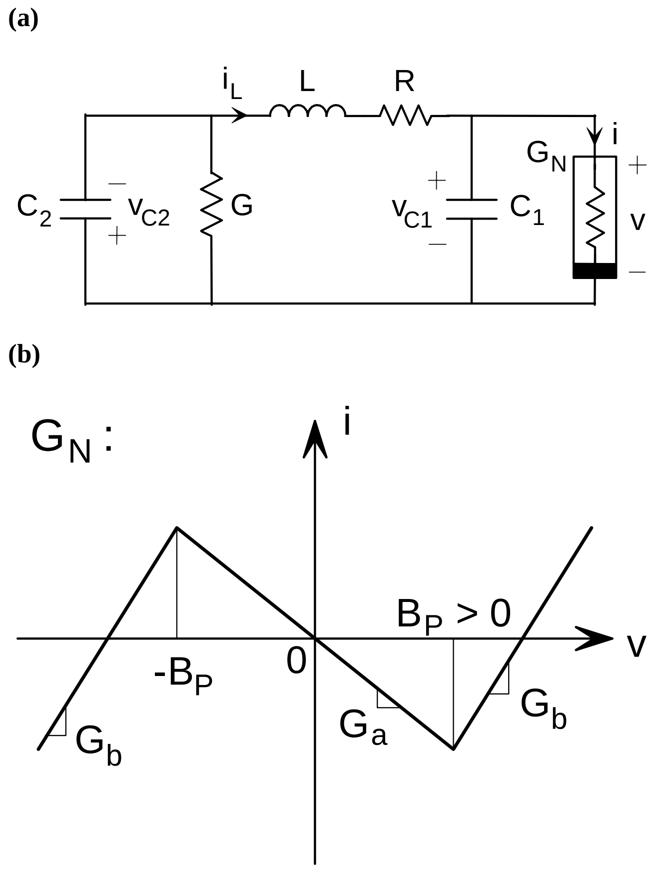
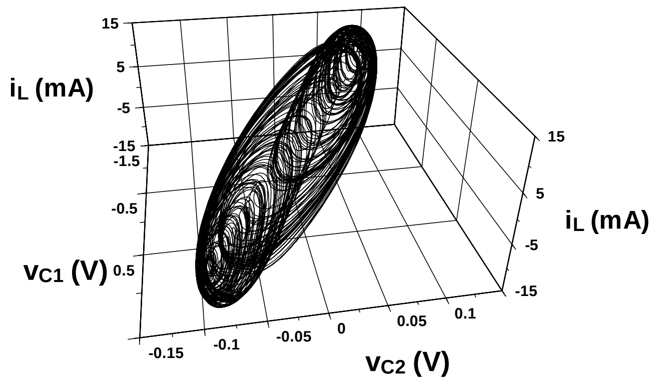


Figure 12: *Chua's canonical piecewise-linear (PWL) circuit [87] chosen as the basis for a high-frequency chaotic communications link.* **(a)** *Circuit diagram.* **(b)** *Representative PWL resistor i-v characteristic.*

been a challenging task because of the frequency-dependent issues that arise in realizing such a broadband oscillator, and since in general there are few systematic approaches to designing such oscillators (but see [84] for one of these exceptions). The details of the background, motivation, progress, and lessons learned from this undertaking can be found in the complementary papers [85, 86], of which the summary given here is abstracted. The first high-frequency oscillator realization centered around the simple baseband circuit shown in Figure 12, known as *Chua's oscillator* [87]. This circuit has become a paradigm for chaos because of its generality and simplicity: the former property coming from its ability

(a)



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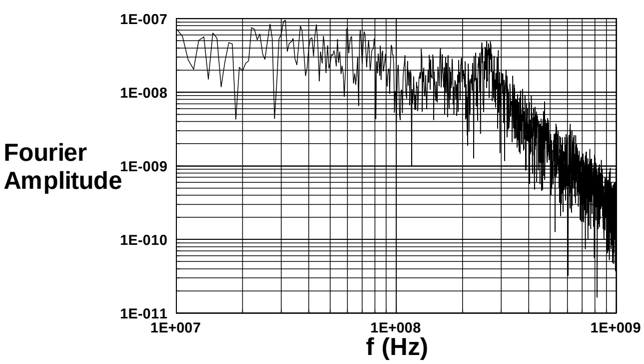
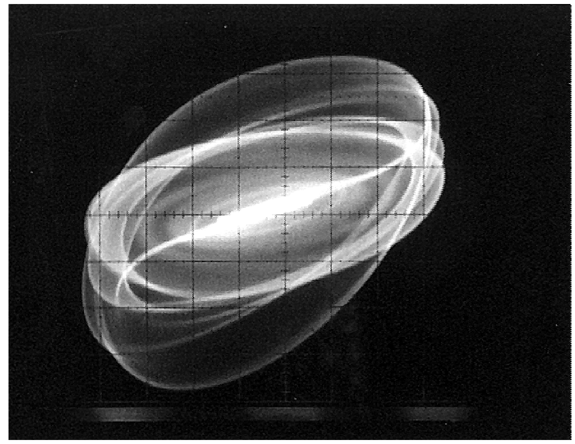


Figure 13: *SPICE simulation of a frequency-scaled microwave version of the circuit in Figure 12.* **(a)** *Three-dimensional phase portrait of strange attractor.* **(b)** *Typical noise-like frequency spectrum for the capacitor voltage v_{C1}.*

(a)



(b)

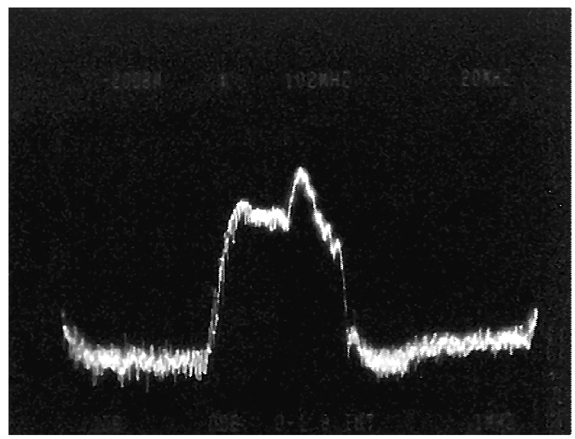


Figure 14: *Experimental views for an autonomous BCO using an inverting-amplifier-based NRG with added delay in the feedback path.* **(a)** *Two-dimensional view of the phase portrait resulting from the two capacitor node voltages (only qualitatively related to actual circuit values).* **(b)** *Power spectrum of a capacitor node voltage.*

to formally realize a whole spectrum of qualitative behaviors, while the latter stemming from the fact that it is third-order (the minimum for a continuous system) and completely linear except for the *piecewise-linear* (PWL) (the simplest form of nonlinearity) resistor G_N. Note from Figure 5(b) that the nonlinear resistor is locally active by having a negative conductance in its middle segment, becoming eventually passive with positive conductance segments in order that it be physically realizable. A simple frequency-scaling of this circuit was simulated using SPICE, with a typical member of its many uncorrelated strange attractors shown in Figure 13. Observe the noise-like frequency spectrum in part (b) of the figure that is one of the hallmarks of chaos.

Note from Figure 12 that the oscillator consists of a passive portion, which is very easy to scale up in frequency, and an active portion needed to realize the PWL resistor, called a *negative-resistance generator* (NRG). It is the NRG that became the challenging part of the implementation effort which we will now outline. First, a novel synthesis of this element was developed that allowed for the straightforward tuning of the breakpoints and slopes of the resistor, an important feature needed for synchronization purposes that would be much more difficult for a non-PWL nonlinearity (such as would be the case for the diode-based oscillators reported in [88]). The high-frequency realization of the oscillators met with several challenges because of the frequency-dependent parasitic and delay effects that naturally arise in the NRG implementation. Several important accomplishments were made in this regard, including the realization of both a 'normal' DC–150 MHz chaotic oscillator, as well a novel *bandpass* chaotic oscillator (BCO) exhibiting 60 MHz chaos centered about a 100 MHz operating frequency. The behavior of the latter oscillator is shown in Figure 14, where a nebula-like strange attractor can be observed in the v_{C1}-v_{C2} phase plane.

After sensitivity studies concerning this oscillator implementation approach found high sensitivity to delays, an alternate realization methodology was invented that was inspired by

the BCO shown in Figure 14. The approach marked a radical shift from an autonomous to a nonautonomous one, with its concomitant advantages of (1) designs are more forgiving with respect to delays and parasitics, since it is not necessary to realize a negative resistance as in the unforced case; (2) the unforced part of the circuit can be second order and hence easily realizable in the microwave regime (using an LC or cavity resonator); (3) it is well known at baseband that the nonautonomous form of synchronization is quite robust against interference in the channel as discussed above (see [30]), a desirable feature for real-world communications applications; and (4) the system naturally provides for phase modulation of the forcing functions, which again translates into a very complicated modulation of the chaotic carrier, hence potentially enhancing message security.

The basic oscillator used in this endeavor was the well-known Duffing oscillator (see [4]) whose characteristic chaotic behavior is shown in Figure 15. This is another well studied classically chaotic system introduced in 1918 (before the word "chaos" came into common usage) to model the hardening spring effect found in many mechanical systems. The nonlinearity here was cubic, corresponding to a symmetrical two-well potential field, and of which a PWL version was invented by the authors whose implementation was called the *Young-Silva chaotic oscillator* (YSCO). This realization strategy turned out to be a breakthrough in the project for which a patent has been applied. Parts (a) and (b) of the figure are simulation results from the dynamical system given by

$$\ddot{x} + \delta\dot{x} - x + x^3 = \gamma\cos(\omega t) =: f(t), \tag{15}$$

where σ represents a coefficient of linear kinetic friction, and γ represents the strength of the sinusoidal forcing function. The behavior has a very rich bifurcation behavior with respect to γ and the frequency ω of the forcing function. Part (c) of the figure shows an oscilloscope display from an experimental baseband series version of the YSCO that can be compared with the simulation in part (a).

Figure 16 shows the success of the YSCO approach by illustrating the phase portrait and frequency spectrum from a series version of the oscillator forced at around 100 MHz and exhibiting about a 120 MHz bandwidth. The behavior was found to be highly insensitive to delay effects, unlike for the previous autonomous oscillators, with tolerance of equivalent phase shifts upwards of 90°. Another unique and useful feature of this implementation approach was that the shape of the frequency spectrum was readily controlled by varying the amplitude and frequency of the forcing function. In this way, for example, the tell-tale signature of the forcing function peak can be essentially removed. A microwave parallel version of the YSCO has also been simulated and fabricated using GaAs FETs. It is projected to be forced at 1 GHz and have a bandwidth near twice that value. Once this oscillator is demonstrated in the laboratory, the next steps will be to develop a synchronization system based on

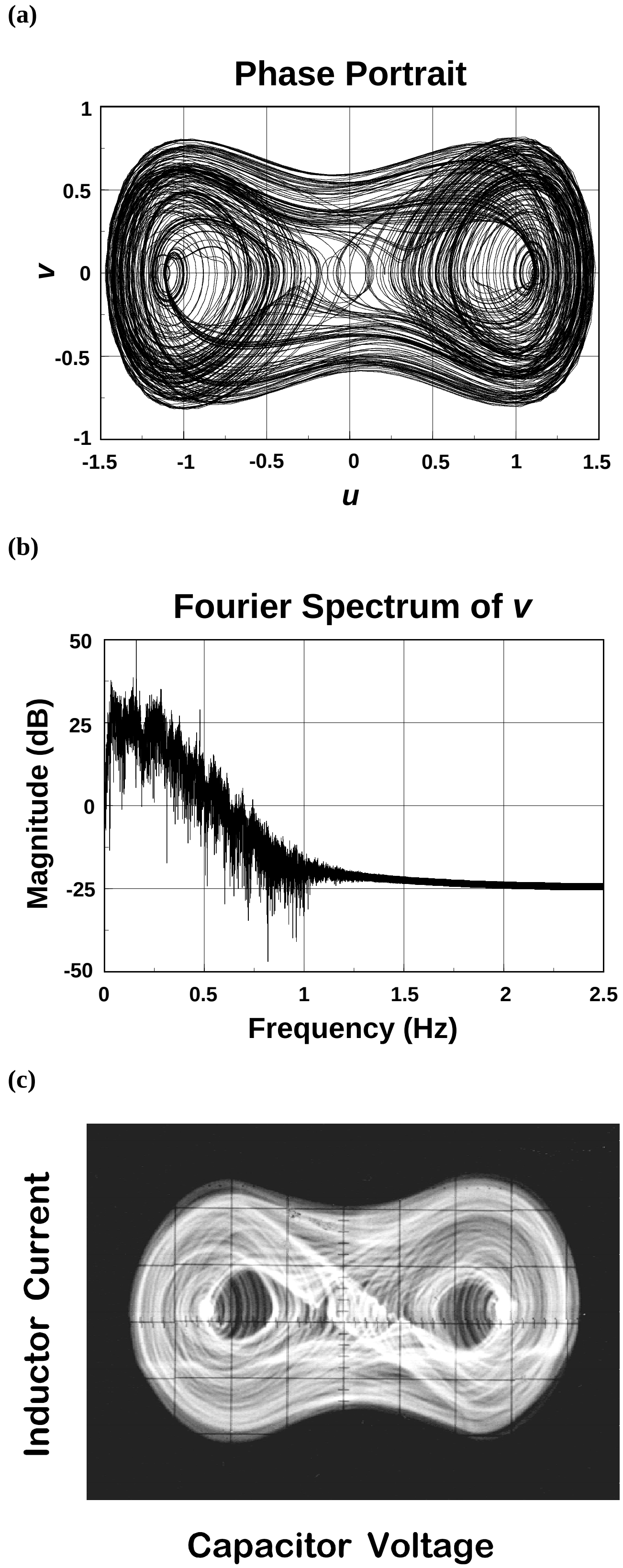
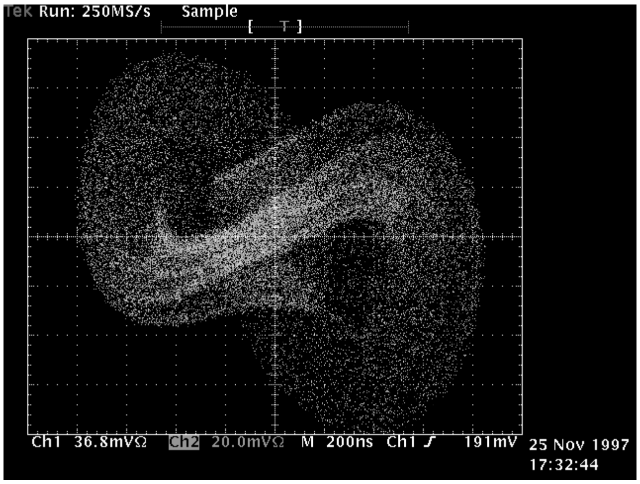


Figure 15: *Simulation of the baseband nonautonomous Duffing oscillator.* **(a)** *Two-dimensional view of the phase portrait.* **(b)** $1/f$*-noise-like frequency spectrum of one of the state variables.* **(c)** *Experimental phase portrait from baseband series YSCO.*

(a)



(b)

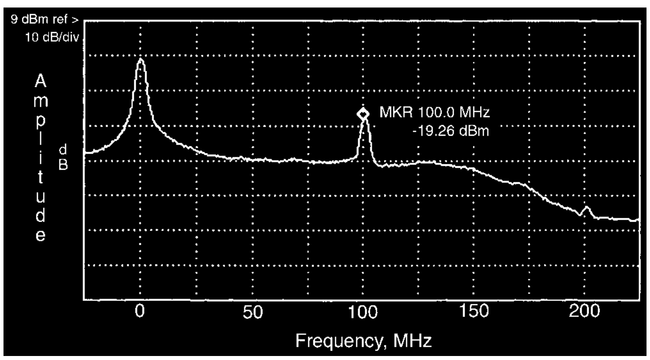


Figure 16: *Experimental results from a YSCO driven at 100 MHz.* **(a)** *Phase portrait that corresponds to the inductor current versus capacitor voltage. The majority of qualitative features found at baseband are preserved in this high-frequency attractor.* **(b)** *Typical capacitor voltage power spectrum, having a significant bandwidth of around 120 MHz.*

them, and then to finally demonstrate chaotic modulation and demodulation.

6. Other Signal Processing Applications

The following abbreviated list provides a set of other interesting applications of chaos that exhibits the richness and state of flux of the field at this time. Relevant references to the literature are provided for further illumination.

(i) The subfield of *control chaos*, which involves the use of adaptive techniques to parametrically control dynamical behavior, holds much promise in applications ranging from enhanced key generators to chaotic signal constellations (see [89] for the latter novel approach to chaotic communications). In particular, these techniques allow for the exploitation of the complicated orbits that chaos can harbor, such as keeping the system on long periodic orbits for key generation; or dividing a strange attractor into several regions, each of which can represent a digital symbol, in chaotic signaling. Also see [63, 90, 91] for general reviews of this growing field. More recently, the area of *anti-control of chaos* has emerged in which chaotic behavior in a given dynamical system is either created, maintained, or enhanced for some useful purpose [92, 93].

(ii) It has been said by the philosopher Spinoza that there is truly nothing random—what appears random really has an underlying structure that has yet to be discovered. With this motivation, the subdiscipline of *deimbedology* has emerged to try to ascertain the dynamical systems that underlie apparently random processes. This would have great practical import, for if common performance-limiting noise processes could be modeled with chaos (e.g. phase noise in oscillators and amplifiers), they could also be subsequently removed adaptively. This so-called *denoising* is already being accomplished with wavelet techniques in such contexts as musical recordings (decoughing) and medical physiology (smart heart pacemakers). Some work along these lines can be found in [94] and [95].

(iii) Another interesting chaotic effect that could have an enormous impact on communications systems is termed *stochastic resonance* that arises, for example, in the circuit shown in Figure 12 at the point where two distinct strange attractors coalesce into one larger attractor. In a generalization of classical resonance, an injected signal with noise exhibits amplification that favors the signal over the noise, the latter resonating with the chaos and being transformed into the signal. In this way, it may be possible to design *nonlinear* amplifiers that *enhance* SNR, a feat totally and equivocally impossible with linear amplifiers. Sources for further study on this and related amplification effects can be found in [96]–[99].

(iv) Based on the fact that different strange attractors do not correlate with each other, analog chaotic versions of CDMA and spread-spectrum systems can also be proposed, with the strange attractor playing the role of a traditional sinusoidal carrier (see [100]). Simulations of these systems suggest that their capacity can be one-and-a-half times that of standard CDMA systems.

(v) Other miscellaneous applications indicating the continued wellspring of potential for chaos include such diverse studies as (1) making music from chaotic circuits using control chaos techniques [101], (2) the interesting effects found in coupled chaotic systems [102], and (3) a continuation of the previous arrangement called *cellular neural networks* (CNN) that perform as analog computers do rapidly perform image processing and other signal processing functions [103].

7. Conclusions and Projections

An introduction to the field of nonlinear dynamics and chaos has been presented, along with a set of representative applications that serve to illustrate the potential impact of chaos on information processing/transfer. The popularity of chaos (and to a somewhat less extent its related fields of fractals and wavelets), is simultaneously a benefit and a detriment to its becoming an established subject worthy of serious consideration. On the one hand, this popularity finally brings an appreciation and interest from the general public for a highly mathematical subject that would normally be ignored by them. On the other hand, the term chaos can become trivialized and misused, diminishing its importance and increasing its skepticism for applications in technical circles, especially traditionally trained engineers. As for all new fields there are growing pains, and this paper has sought to lessen these by providing an objective and technical assessment of the state of the field and its application potential. An excellent outlook on the whole of nonlinear science can be found in the special theme issue [104].

It should be noted that the practice of applied chaos is the least developed among the other two mentioned areas of applied nonlinearity. In fact, there are currently commercial software and hardware products available using the principles of fractals and wavelets, but this cannot be said for chaos. To be more specific, the following two representative examples illustrate this point:

(i) *Fractal interpolation* and wavelet methods have been fruitfully applied to information/image compression, with such examples as the Microsoft® Encarta™ CD and the recently adopted FBI fingerprint database system [105]. These applications are based on the converse of the notion that simple dynamical systems can produce complex behavior—that is, given some realistic signal/image, can one systematically distill a simple prescription by which this information can be represented faithfully? If so, then the prescription is transmitted instead of the real information, thereby providing the compression (which can typically range from factors of 20 for fingerprints to hundreds for fractally-interpolated images).

(ii) A potential new paradigm for communications signaling is currently under development using wavelets, providing generalized redundancy and orthogonality that is effective against such contexts as rapidly changing and unknown channels [106]. The idea here is to exploit one of the powerful advantages of wavelets: the flexibility to tailor them to a given problem. In this context, this allows for the versatile design of a multi-dimensional signal constellation that is optimal for a given set of channel characteristics. An additional benefit of this approach is that the modulation and demodulation operation is a straightforward up/down sampling process that is a natural for digital implementation. It is highly probable that wavelet communications will become a major form of information transfer in the not-to-distant future.

It is hoped that this paper will remedy this situation by inspiring research and development engineers to seriously consider transforming chaos-based techniques into marketable products. Finally, it should be noted that this paper has only provided a mere sampling of the open frontier of applied nonlinearity, leaving out other exciting areas such as the use of *solitons* for dispersionless long-haul fiber optical communications, and nonlinear system modeling and distortion-compensation techniques to enhance the performance of satellite communications systems; not to mention the equally vigorous activity taking place in the area of medical physiology that stems from nature's inherent nonlinearity. In order to appreciate this richness, the reader is encouraged to visit the interesting Internet WWW sites [107]–[112] on nonlinearity, as well obtain and watch the video [113] displaying the deep beauty of nonlinear dynamics.

Acknowledgment

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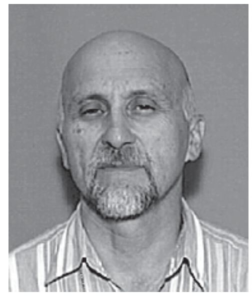

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